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Holography and phenomenology

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Abstract: We examine various aspects of the conjectured duality between warped AdS_5 geometries with boundary branes and strongly coupled (broken) conformal field theories coupled to dynamical gravity. We also examine compactifications with 5-d gauge fields, in which case the holographic dual is a broken CFT weakly coupled to dynamical gauge fields in addition to gravity. The holographic picture is used to clarify a number of important phenomenological issues in these and related models, including the questions of black hole production, radius stabilization, early universe cosmology, and gauge coupling unification.

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Contents

1. Introduction

The lesson from the AdS/CFT correspondence [1, 2, 3] is that a gravity theory on AdS_5 is equivalent to a strongly coupled 4D CFT. However, when all the symmetries are fully realized, the physical equivalence between the theories is slightly obscure.

Normally, we think of two theories as being equivalent if they make identical predictions for all physical processes, such as the spectrum or S-matrix elements. However in standard AdS/CFT, the theories on both sides are a little removed from our usual intuition; on the 4D side, CFT's do not have S-matrices or a "spectrum", and the same is true on the AdS side. The equivalence takes on a slightly more abstract form; for every CFT operator, $\mathcal{O}$, there is a corresponding bulk field ϕ. Given any boundary condition for ϕ, $\phi_0(x)$, at the 4D boundary of AdS, there is a unique solution of the gravity (or more generally the full string) equations of motion in the bulk; let $\Gamma[\phi_0(x)]$ represent the SUGRA action (or more generally full string effective action) of this solution. The AdS/CFT correspondence tells us that

$$\left\langle \exp\left(-\int d^4x \phi_0 \mathcal{O}\right)\right\rangle_{\mathrm{CFT}} = \exp(-\Gamma[\phi_0])\,. \tag{1.1}$$

This correspondence does contain all the physical information we can extract out of the theories on both sides; however both sides are a little unfamiliar.

In this respect, the addition of the Randall-Sundrum "Planck brane" [4] and/or a "TeV brane" [5], which chop off parts of the AdS, is very interesting. These constructions have very simple holographic interpretations, which as we will see illuminate many aspects of the physics and phenomenology of these models. It is also nice that the addition of the Planck and/or TeV branes act as regulators, and allows for a more intuitive understanding of the holographic equivalence itself. For instance, adding a "Planck brane" to the gravity side, and making the four dimensional graviton dynamical by integrating over the boundary conditions, corresponds to putting a UV cutoff $\lesssim M_{Pl}$ on the CFT and adding 4D gravity. Now, the field theory is still conformal and so there is still no spectrum or S-matrix, but we now have gravity to act as a probe on both sides of the correspondence. We can now ask the same physical questions on both sides, such as what is the the leading correction to Newton's law between two masses on the Planck brane? On the gravity side, this calculation is weakly coupled, and comes from the exchange of the continuum of KK gravitons. On the CFT side, we compute the CFT loop correction to the 4D graviton propagator, which is a strongly coupled calculation but is fortunately totally determined by conformal invariance. The fact that the force law is identical whether one thinks in terms of living on a Planck brane in an infinite fifth dimension or being gravitationally coupled to a CFT in 4D is a strikingly physical illustration of holography. The situation becomes even clearer with the addition of a "TeV brane", which corresponds to some deformation of the CFT leading to a breakdown of conformal invariance in the IR. Now, we really do have particles and S-matrix elements, so the statement of the holographic equivalence between the broken CFT and the gravitational picture is the familiar one, namely the two theories have identical spectra, identical S-matrices.

2. The holographic intepretation of RS2

Throughout this paper, we will assume that the AdS/CFT correspondence can be extended to tell us that *any* 5D gravity theory on AdS_5 is holographically dual to *some* strongly coupled, possibly large N, 4D CFT. We will see that this assumption satisfies various consistency checks, at least qualitatively in the course of this paper. In Poincaré co-ordinates, the metric of AdS_5 is

$$ds^2 = \frac{L^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)\,. \tag{2.1}$$

Notice that a rescaling $z \to \lambda z, x^\mu \to \lambda x^\mu$ leaves the metric invariant; translations in $\log z$ correspond to 4D scale transformations. Loosely speaking, the fifth dimension z is encoded in the 4D theory as the RG scale, with larger z/L corresponding to the infrared in the CFT. The self-similarity of the 5D background is then interpreted as the conformality of the 4D theory.

Beginning with this correspondence, we can start deforming one side in some way and ask what the deformation means on the other side. We can start by modifying the gravity side. One thing we can do is add the Randall-Sundrum "Planck brane". We will review recent results indicating that the duality applies in this case [6, 7].

A convenient way of describing this is to have a metric which is AdS for $z > z_c$, and simply reflect this space about z_c for $z < z_c$. The resulting metric is, after some obvious variable changes and coordinate rescalings,

$$ds^2 = \frac{1}{(1+|z/L|)^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)\,. \tag{2.2}$$

There are clearly discontinuities in the curvature at $z = 0$, but they are attributed to a delta function brane at $z = 0$, with a specific tension finely tuned to the bulk cosmological constant. This is the "Planck brane". Studying the metric fluctuations about this background reveals a zero mode graviton localized to the brane at $z = 0$, with 4D Planck mass M_4 related to the 5D Planck mass M_5 via

$$M_4^2 \sim M_5^3 L\,. \tag{2.3}$$

There is also a continuum of KK modes in the spectrum, which however has small couplings to sources on the Planck brane. The continuum modes give a correction to the newtonian potential between two test masses m_1, m_2 separated by a distance R on the brane of the form

$$V(r) = \frac{m_1 m_2}{M_4^2}\left(\frac{1}{r} + \frac{L^2}{r^3}\right). \tag{2.4}$$

What does this deformation correspond to in the gauge theory? We are modifying the space away from pure AdS at small z, which corresponds to the ultraviolet

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in the gauge theory. The most natural guess is that we have taken the CFT, put some sort of UV cut-off near M_4, and added 4D gravity to the theory. The precise structure of the "Planck brane" is reflected in the precise nature of the UV cut-off in the dual theory. So, the deformation of AdS/CFT to the present case seems to suggest that *the RS2 theory with localized gravity in an infinite fifth dimension is dual to a strongly coupled CFT with a cutoff + 4D gravity* [6, 7]. This conjecture passes one immediate check. From the 4D viewpoint, the leading correction to the Newton potential comes from the 1-loop correction to the graviton propagator, which is schematically, in momentum space

$$\frac{1}{p^2}\langle T(p)T(-p)\rangle\frac{1}{p^2}\,, \tag{2.5}$$

where T is the CFT energy momentum tensor. Despite the fact that the theory is strongly coupled, this $\langle TT\rangle$ correlation function is fully determined by conformal invariance

$$\langle T(x)T(0)\rangle \sim \frac{c}{x^8} \to \langle T(p)T(-p)\rangle = cp^4\log p^2\,, \tag{2.6}$$

where c is the central charge. When transformed back into position space, this gives the correct $1/r^3$ correction found from the gravity side [6, 7] (see also [8, 9]).

If there are fields (say the SM) localized on the Planck brane, then in the 4D theory they interact with the CFT both through 4D gravity, and potentially also through higher-dimension operators suppressed by M_4.

We could have of course started instead from the 4D side, and deformed the theory by gauging some of its global symmetries. The most natural symmetry to gauge is the Poincaré symmetry of the 4D theory, i.e. add 4D gravity with some Planck scale M_4. Since this corresponds to a big modification at high energies $\sim M_4$, on the dual side we should expect some breakdown of AdS at small z, and this is the "Planck brane". Actually, we need to be a little more precise about "cutting off" the CFT. It is important to recognize that from the weakly coupled gravity picture, we see that the field theory remains conformal in the IR in the presence of the cut-off and gravity. Obviously a random way of cutting off the CFT and adding gravity will not guarantee that the low energy theory is conformal, for instance mass terms may be generated that destroy the conformal invariance. Consider as an example weakly coupled $\mathcal{N} = 4$ SYM, and add ordinary 4D gravity (not supergravity). Then, at two loop order, the 6 scalars pick up quadratically divergent masses near the cutoff M_4, and the low energy theory is certainly not conformal. Had we added even just $\mathcal{N} = 1$ supergravity, the scalar masses would be protected and the low energy theory would continue to be conformal. What is special about the way that conformal invariance is broken in the holographic dual of RS2 is that the breaking of conformal invariance shows up only in Planck *suppressed* operators, not in mass terms. If we want a completely non-supersymmetric example, we need a conformal theory where, even in the presence of gravity, there are no relevant deformations which can not be prohibited by symmetries. For instance, a purely fermionic conformal gauge theory with fermion masses prohibited by chiral symmetries might have this property.

We must remark that the holographic correspondence in the way described above is only valid at energies $E \ll L^{-1}$. For instance, on the gravity side, at distances $r \ll L$, the gravitational potential turns over to the 5D ($1/M_5^3 r^2$) form, or more generally to the $(4+n)$D form $1/M_{4+n}^{2+n} r^{2+n}$. The CFT +4D gravity can not possibly reproduce these results. This is expected from the full duality between the CFT and string theory on $AdS_5 \times S_5$, the 5D gravity/4D CFT correspondence is only valid at distances larger than the radius of the S_5 which is L.

2.1 Wilsonian renormalization and holography

Moving the position of the UV brane from $\Lambda = 1/L$ to $\Lambda' < \Lambda$ corresponds to integrating out the degrees of freedom with energy in between the two cutoffs. This correspondence has been analyzed in several papers [10]–[13]. Specifically, ref. [13] analyzes the case where $M_{Pl} < \infty$, where gravity is dynamical. In this subsection, therefore, we will be brief and only point out to one simple check of this correspondence. In the RS background, using the metric in eq. (2.1), the zero-mode of the 5-d graviton reads

$$g_{55}(z,x) = \frac{L^2}{z^2}\,, \qquad g_{5\mu}(z,x) = 0\,, \qquad g_{\mu\nu}(z,x) = \frac{L^2}{z^2}\gamma_{\mu\nu}(x)\,. \tag{2.7}$$

We set the UV brane at $z_c = L$. The 4-d Planck mass is obtained by substituting the zero mode into the 5-d Einstein action

$$\begin{aligned} S &= M_5^3 \int_L^\infty dz \int d^4x \sqrt{-g} R^{(5)}(g) = M_5^3 \int_L^\infty dz \frac{L^3}{z^3} \int d^4x \sqrt{-\gamma} R^{(4)}(\gamma) \\ &= \frac{M_5^3 L}{2} \int d^4x \sqrt{-\gamma} R^{(4)}(\gamma). \end{aligned} \tag{2.8}$$

This equation gives the result mentioned above: $M_4^2 = M_5^3 L/2$. If one adds an IR brane at $z = 1/\mu \gg L$, as in RS1 [5], then the 4-d Planck constant is:

$$M_4^2 = M_5^3 \int_L^{1/\mu} dz \frac{L^3}{z^3} = M_5^3 L^3 \left(\frac{1}{L^2} - \mu^2 \right). \tag{2.9}$$

These results for the 4-d Planck mass are consistent with the interpretation of RS2 as a CFT with cutoff $1/L$ coupled to gravity, and also suggest that the holographic dual of RS1 is a CFT broken at scale μ coupled to gravity. Both in RS1 and RS2 the Planck scale is fully induced by the (broken) CFT. Notice that both the scale dependence in eq. (2.9) and the numerical coefficient $M_5^3 L^3$ are consistent with this interpretation. Indeed, in field theory, the inverse Newton constant induced by a CFT broken at scale μ and with cutoff $1/L$ is $c(L^{-2} - \mu^2)$, where c is uniquely

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determined by the central charge of the CFT; the holographic computation of c using 5-d supergravity gives precisely $M_5^3 L^3$ [14].

Generically, quantum gravity may not be fully induced by the CFT. This happens when a 4-d Einstein action is introduced as a boundary term on the UV brane, i.e. when the term $M_0^2 \int d^4x \sqrt{-g} R^{(4)}(g)|_{z=L}$ is added to the 5-d Einstein action. In this case, the 4-d Planck scale is given by

$$M_4^2 = M_0^2 + M_5^3 L^3 (\Lambda^2 - \mu^2)\,, \qquad \Lambda = \frac{1}{L}\,. \tag{2.10}$$

This boundary term is obviously necessary if we want to integrate out UV degrees of freedom by moving the Planck brane from $z = L$ to $z = 1/\Lambda' > L$. To keep the physical 4-d Planck mass invariant we must change M_0^2 to $M_0^2 + M_5^3 L^3 (\Lambda^2 - \Lambda'^2)$. In [15] for example, where there is an embedding of the RS-like background into string theory, M_0^2 does not vanish. Sending $M_0 \to \infty$ leaves us with the 5D holographic dual of just a cut-off CFT, *without* gravity.

2.2 Qualitative holography

Holography tells us that the z direction of gravity theory can be interpreted as the RG scale of the 4D theory. It is then amusing to consider what various "bulk" phenomena look like in the 4D picture. Of course this topic has been extensively investigated in the infinite AdS case and has also been nicely analyzed in the RS case [16], so we will briefly consider one bulk process by way of illustration. Imagine a graviton, starting at a given point (x, z_0) and aimed straight at the Planck brane where the SM fields live. After the time required to traverse the distance to the Planck brane (which in this background metric is $T \sim z_0$), the graviton bangs into it and creates e.g. photons. For times $t < T$ the graviton has not hit the Planck brane yet and so there is no photon production in the classical limit. How do we understand this from the 4D side? By holography, a point particle localized at some point (x, z) in the bulk must correspond to a shell-like configuration of the CFT of size z centered around x. If the bulk particle is moving at the speed of light towards the Planck brane, from the 4D side the size of the CFT lump is decreasing at the speed of light. When the shell is of size M_4^{-1}, the standard model fields will couple to the conformal field theory modes via the gravitational coupling. Photon production then happens after the time it takes for the lump to decrease from size z_0 to M_4^{-1}, which is just $\sim z_0$, in agreement with the 5D picture.

2.3 The strong coupling "problem"

Purely from the gravity side, the continuum KK modes of RS2 are somewhat mysterious. While we see them upon linearizing about the background at quadratic order, their self-interactions are divergent due to the strong coupling region $z \to \infty$. At large z, it is not clear that we can think of the KK modes as the physical particles in the theory because they are not weakly coupled. But on the other hand, quantities which inclusively sum over the KK modes, such as the correction to the Newton potential, seem perfectly sensible. From the holographic point of view, this is obvious: there are no "particles" in the CFT. However, inclusive questions involving an external probe (like 4D gravity in this case) can give perfectly sensible answers.

That there is no strong coupling problem can be understood directly from the gravity side from the argument in [4]. In order to most clearly understand the technical issues, it is most convenient to work in momentum space for 4D and position space in z. For a process on the Planck brane to probe strong coupling in the bulk, we need to use the brane-bulk propagator $P(p_4^2, z)$. The crucial point is that for large enough z,

$$P(p_4^2, z) \sim e^{i\sqrt{p_4^2}z} \,. \tag{2.11}$$

All amplitudes involve integrals of propagators convoluted with z dependent couplings in the bulk, but the couplings never grow worse than powers of z, and therefore the rapid exponential die-off or oscillation of p ensures that the contributions from regions with $z > 1/p_4$ are negligible. Processes with external momenta p_4 on the brane are insensitive to what happens at $z > (1/p_4)$ in the bulk. This obliterates the strong coupling concern in RS2. It tells us more; even if the geometry were to change to a different AdS or deviate from AdS altogether for z bigger than some z_c, for momenta $p_4 > 1/z_c$ on the Planck brane we would not detect measurable physical effects.

All of this has a natural holographic interpretation: external graviton momenta $\sim p_4$ are only sensitive to the behavior of the theory at energies $\sim p_4$ and are in particular insensitive to anything that happens at far lower energies; e.g. masses can be neglected at very high energies.

3. Holography and RS1

3.1 Generalities

Putting a "TeV brane" into the theory represents a departure from AdS at large z and must therefore correspond to a breakdown of conformal invariance in the IR in the 4D side. Notice that the conformal field theory is badly broken in the IR; all remnant of conformal invariance disappears beneath this scale. Furthermore, the low-energy field theory is weakly coupled and hence should not have a weakly coupled gravity description. This holographic description is consistent with the form of the induced Newton constant we obtained earlier in eq. (2.9). Now that the conformal invariance has been broken, we expect to get physical particles that we can asymptotically separate and so we can have S-matrices. So, the statement of the holographic equivalence is simply the intuitive one; the spectra are identical and the S-matrices are the same! In particular, the KK gravitons in the gravity side can be interpreted in the 4D theory as resonances. Of course, since the 4D theory is strongly coupled we could have never guessed that the low-lying spectrum of excitations includes a tower of spin-2 particles. For this, the gravity description is more useful, at least over the energy range where it is weakly coupled. In order for the gravity description to be weak, we need that $LM_5 > 1$; note that all the KK graviton masses are quantized in units of μ and they become more and more narrow as M_5L increases. So, if we live on the TeV brane, the 5D gravity description is weakly coupled, and gets strongly coupled for $E > \tilde{M}_5 \equiv M_5\mu L$. Actually, since the first KK modes have mass $\sim \mu$, the theory does not look 5D until $E > \mu$, so that the 5D gravity picture is useful in the regime $\mu < E < \tilde{M}_5$. In [5], M_5L is chosen to be somewhat bigger than one to for a reliable weak gravity description (since the hierarchy was being generated by the exponential warp factor there was no need to make M_5L very large). We can however comfortably imagine M_5L to be as big as one or two orders of magnitude.

In RS1, the SM fields live on the TeV brane. We now consider the question of how they to be interpreted on the 4D side. In this analysis, we will assume there is a valid description at energies above the TeV scale. In this case, since from the 5D side we see that the SM states become strongly coupled to the KK gravitons above $\tilde{M}_5$, and since from the 4D side these KK gravitons are CFT bound states, we cannot think of the SM states as "spectators" to the strong dynamics. Rather, the SM must emerge as bound states out of the strong dynamics. So, RS1 does not literally correspond to a technicolor theory (we will see what does when we discuss gauge fields in the bulk). Rather, it is a theory where the SM becomes embedded in a strong CFT above the TeV scale. As such, it shares features in common with the proposal of [17] to solve the hierarchy problem by embedding the SM in a conformal theory at a TeV. There are some important differences, however: the models of [17] were not at strong coupling, and furthermore the explicit non-SUSY models they considered were obtained by orbifolding conformal SUSY models, and were therefore only conformal in the large N limit, requiring a huge $N \sim 10^{30}$ to stabilize the weak scale [18]. As discussed in the introduction, it is a nontrivial thing to break conformal symmetry while not reintroducing quadratic divergences. The cutoff of RS1 must do this, as is readily seen from the gravity side. Generic cutoff procedures will not work.

Finally, an exercise in qualitative holography very similar to what we did for RS2 is instructive. Suppose we aim a graviton from e.g. the Planck brane towards the TeV brane. From the gravity point of view, it takes some time $\sim \mathrm{TeV}^{-1}$ for the graviton to reach the TeV brane and produce SM particles. From the CFT point of view, we have a spherically symmetric lump of CFT that starts with a small size and grows at the speed of light. Only when it reaches a size of order TeV^{-1}, however, can this lump know about the breaking of the CFT and create SM particles, and this takes the same time TeV^{-1} as in the gravity picture.

3.2 "String theory" at a TeV

Of course, all of this physics is in principle contained in the strongly coupled field theory, and for many processes involving energies far above $\tilde{M}_5$, in the 4D picture the breaking of conformal invariance is unimportant and thinking in terms of a strongly coupled CFT is most useful. For instance, since we see that the theory gets strongly coupled at $\tilde{M}_5$ from the 5D side, we would conclude that we hit "string theory" at $\tilde{M}_5$. This is certainly true; we hit string theory propagating on this particular AdS background. It's just that string theory on this background behaves quite differently from what we normally think of as string theory in flat space; AdS/CFT tells us that this string theory is really the "QCD string" of a strongly coupled 4D CFT. Let us try to better understand physics at energies above $\tilde{M}_5$. For instance, what happens as we heat the system to temperatures $T \gg \tilde{M}_5$? One might think, from a naive expectation of string theory, that there would be a Hagedorn limiting temperature. However, just as in QCD, no such limiting temperature exists. At high energies the theory is described by a (strongly coupled) 4D CFT, and despite the strong coupling, conformal invariance is enough to tell us the free energy is the usual one for radiation $F \sim T^4$. Why do not we get the Hagedorn limiting temperature? After all, from the 5D viewpoint, we certainly have massive string modes in the bulk, and their wave functions will be localized on the TeV brane. We may also have open strings stuck to the TeV brane. So why do not we get the usual Hagedorn exponential density of states? In the analogous case in QCD, what happens is that the hadrons become so broad that they cannot be thought of as particles any more, and instead we see quarks and gluons at high temperature. Exactly the same thing happens here. These string modes are just broad bound states of the CFT, and do not give rise to a Hagedorn spectrum of physical states.

It is not surprising that a strongly coupled 4D theory should have broad resonances; what is more interesting is that we get narrow resonances: the spin 2 KK gravitons! Note that the KK gravitons are narrow when $M_5 L$ is large, which corresponds to large N in the 4D theory, so this is perhaps not unexpected. We get roughly $M_5 L$ of these resonances starting at μ and ending at $\tilde{M}_5$, above which they too become too broad to be called particles. So, following the change in free energy as the system is raised first above temperature μ to just beneath $\tilde{M}_5$, we have

$$F^{-}(T) = T^5 \mu^{-1}\,, \qquad \mu < T < \tilde{M}_5\,. \tag{3.1}$$

Note again that in this intermediate region $\mu < T < \tilde{M}_5$, the free energy looks that of a 5D theory. However, for temperatures $T > \tilde{M}_5$, the free energy should be same as the pure CFT at temperature T, which is $F^{+}(T) \sim cT^4$ where c is the central charge, $c = (M_5 L)^3 \equiv \tilde{M}_5^3 \mu^{-3}$ [19]. Note that there is a transition in F between $T < \tilde{M}_5$ and $T > \tilde{M}_5$:

$$F^{-}(\tilde{M}_5) = \tilde{M}_5^4 (M_5 L)\,, \qquad F^{+}(\tilde{M}_5) = \tilde{M}_5^4 (M_5 L)^3\,. \tag{3.2}$$

The difference $F^+(\tilde{M}_5) - F^-(\tilde{M}_5)$ signals a deconfining phase transition in the broken CFT at $T = \tilde{M}_5$.

3.3 Black holes at a TeV

One of the features we expect of a quantum gravity theory is that, when particles are scattered at energies high above the Planck scale, semi-classical black-holes are formed. Once they are made, these black holes decay via Hawking radiation. In a usual flat space background, for $E \gg M_{Pl}$, as long as the impact parameter of the collision is less than the Schwarzschild radius R_E of the would be black hole with the c.o.m. energy E, a black hole will form with essentially unit probability. The production cross-section is then simply the geometrical area

$$\sigma(E) \sim R(E)^2 . \tag{3.3}$$

In 4 flat dimensions, $R(E) \sim E/M_{Pl}^2$, so $\sigma \sim E^2/M_{Pl}^4$: the cross-section grows indefinitely.

What is the situation in RS1? For the moment, let us assume $M_5 L \sim 1$ for simplicity. Also, since we are interested in creating 5D black holes centered on the TeV brane, let us ignore the Planck brane, which is essential for 4D gravity but has negligible effect for 5D black holes. Of course, whatever 5D black hole production and decay is, it corresponds to some physics in a strongly-coupled 4D field theory, but once again the real question is a physical one: what do we see when we scatter SM particles at energies $E \gg$ TeV? Recall that from the broken CFT point of view, the SM particles are bound states of size TeV^{-1}. Scattering SM particle at energies $\gg$ TeV is much like scattering protons at energies $\gg \Lambda_{QCD}$. If the impact parameter is bigger than TeV^{-1}, nothing happens! This is already a dramatic difference from the usual flat space intuition, but it is equally evident from the gravity side. Recall that in the flat space case, the only reason that two particles at high impact parameter can form a black hole is that they attract each other through the long-range gravitational force. However, the long-range gravity between two particles on the TeV brane is exponentially small, due to the gap in the KK spectrum. So, while it is true that a mass of, say, 10,000 TeV would form a black hole of Schwarzschild radius 100 TeV^{-1} centered on the TeV brane, two particles of energy 5000 TeV impinging on each other with this large an impact parameter would simply fly past each other without forming a black hole. Only if the impact parameter is less than $\sim$ TeV^{-1} can they collide and form the black hole, but the cross-section for doing this never gets bigger than $\sim$ TeV^{-2}.

On the other hand, if we consider the case $M_5 L \equiv \tilde{M}_5 \mu^{-1} \gg 1$, say ~ 10, then black holes of size between $\tilde{M}_5^{-1}$ and μ^{-1} are essentially the flat space black holes, and since they are smaller than μ^{-1}, the gap is irrelevant and they are made with the usual flat space cross section, which grows with energy. Notice that in all cases, the black holes are some excitation of the CFT and, since the CFT is broken, will eventually decay to the lightest CFT bound states, which are the SM particles. To the extent that $M_5 L$ is large, this should mimic the thermal radiation one expects from the 5D gravity picture.

3.4 The radion

In the original RS1 model, by fine-tuning the tension of the negative tension brane, a static background geometry was obtained, with the separation between the branes determined by the expectation value of a modulus. This "radion" degree of freedom can be identified with perturbations of the metric of the form

$$ds^2 = \frac{1}{(1+|z|/L)^2}[dx_4^2 + T^2(x)dz^2]\,. \tag{3.4}$$

Since all mass scales on the negative tension brane are set by T, the radion couples conformally to all states on the IR brane.

The radion wave function is peaked on the TeV brane, and all of its couplings are 1/TeV suppressed [5]; indeed this mode survives even if the Planck brane is removed to infinity. As such, it must be an excitation of the dual 4D theory. But how can the scale of conformality breaking in the dual theory be undetermined? We are not used to this in non-SUSY theories, but SUSY theories almost always have moduli spaces, and if they are conformal they are usually only conformal at the origin of moduli space. Moving along the moduli space then gives a soft breaking of conformality.

In any case, given that the radion exists even in the absence of the Planck brane, we can identify it in standard AdS/CFT examples. For instance, take $\mathcal{N}=4$ SYM, and move along the Coulomb branch in such a way as to break the nonabelian symmetry completely. Now, on the gravity side, moving along the Coulomb branch causes a departure from AdS at large z, and the 5D metric develops a naked singularity. This singular region plays the role of the TeV brane in this case. Examining the perturbations on the gravity side indeed reveals an exactly massless "radion" with wave function peaked on the TeV brane.

The corresponding CFT mode is easily identified. Among the moduli in the $\mathcal{N}=4$ theory, one modulus T can be taken to set the overall scale of all the adjoint vevs, and the others can be taken to be dimensionless. Since this is the only source of conformal violation in the theory, every mass parameter will be proportional to T, which is just the same as the conformal coupling of the radion.

Now we can discuss the holographic interpretation of the Goldberger-Wise stabilization mechanism [21]. They consider a bulk scalar field ϕ of mass m^2, with (m^2L^2) somewhat small (throughout the rest of this section, we will work in units where $L=1$). They also put large potentials on the Planck and TeV branes that energetically force $\phi = v_{UV}$ on the UV brane and $\phi = v_{IR}$ on the TeV brane with $v_{UV} \neq v_{IR}$. (The presence of a large potential for ϕ on the branes also gives a large positive mass[2] to the zero mode of ϕ in the 4D theory, regardless of the sign of m^2). Minimizing the energy stored in ϕ in the bulk then fixes the interbrane separation. In detail, the bulk scalar equation of motion is (neglecting the back-reaction of ϕ on the metric)

$$(\partial^2 + m^2)\phi = 0 \to z^2\phi'' - 3z\phi' - m^2\phi = 0\,. \tag{3.5}$$

A trial solution of the form

$$\phi(z) \sim z^{\Delta}\,, \tag{3.6}$$

gives a solution as long as Δ satisfies

$$\Delta(\Delta - 4) = m^2\,. \tag{3.7}$$

The most general solution for ϕ in the bulk is then of the form

$$\phi(z) = Az^{\Delta_-} + Bz^{\Delta_+}\,, \tag{3.8}$$

where A, B are constants that will have natural interpretations in the holographic picture. Fixing $\phi(z = z_{UV} = 1) = v_{UV}$ and $\phi(z = z_{IR}) = v_{IR}$ then allows us to solve for A, B. When m^2 is small and z_{IR} is large, we can approximate $\Delta_+ = 4 + \frac{m^2}{4}, \Delta_- = -\frac{m^2}{4}$ and

$$A = v_{UV}, \qquad B = \frac{1}{z_{IR}^4}\left(v_{IR} - v_{UV}z_{IR}^{-\frac{m^2}{4}}\right). \tag{3.9}$$

The energy stored in the ϕ field

$$V(z_{IR}) = \int_1^{z_{IR}} dz \frac{1}{z^5}\left(z^2\phi'^2 + m^2\phi^2\right) \tag{3.10}$$

is easily computed; the leading contribution in the expansion in m^2 is simply

$$V(z_{IR}) = 4z_{IR}^4 B^2\,. \tag{3.11}$$

This potential is minimized when $B = 0$, which determines z_{IR} via

$$v_{IR} = v_{UV}z_{IR}^{-\frac{m^2}{4}} \to z_{IR} = \left(\frac{v_{IR}}{v_{UV}}\right)^{-\frac{4}{m^2}} \tag{3.12}$$

and it is evident that an exponential hierarchy can easily be generated. Note that the GW mechanism works whether m^2 is positive or negative [22]. In order to generate a large hierarchy, we need $v_{UV} > v_{IR}$ for $m^2 > 0$ and $v_{UV} < v_{IR}$ for $m^2 < 0$.

Now for the holographic interpretation. In the 4D picture, the bulk scalar field ϕ is associated with perturbing the CFT by the addition of an operator $\mathcal{O}_\phi$, of dimension Δ_+. The quantity Az^{Δ_-} is the running coupling constant for this operator at scale z, or in other words at the UV cutoff, the lagrangian is perturbed as

$$\mathcal{L}_{CFT} \to \mathcal{L}_{CFT} + AO_\phi\,. \tag{3.13}$$

The quantity B is the vev of the operator O_ϕ:

$$B = \langle O_\phi \rangle \,. \tag{3.14}$$

When m^2 is small, O_ϕ is marginally relevant (for $m^2 < 0$) and marginally irrelevant (for $m^2 > 0$).

Consider first the case where $m^2 < 0$. Then, the coupling starts at v_{UV} and grows slowly in the infrared, until it finally hits a critical value v_{IR} where it triggers a complete breakdown of conformality (reflected as the IR brane in the gravity picture). This is the familiar picture of dimensional transmutation; a large hierarchy being generated by a marginal coupling getting strong in the IR. Since B vanishes, in this vacuum $\langle O_\phi \rangle = 0$. The case $m^2 > 0$ is less standard, but equivalent in many respects. Here, it is better to think of generating the Planck brane (in some sense, these two theories are equivalent by interchanging $z \to 1/z$). In this case, the coupling starts at v_{IR} and increases slowly in the UV, until when it gets larger than a critical value it triggers the breakdown of conformality in the UV. Notice for this interpretation, it is critical that conformality is broken via mass suppressed operators in the UV, but mass operators in the IR. Again in this vacuum $\langle O_\phi \rangle = 0$.

We can also see what corresponds to changing the interbrane separation. Suppose we move the IR brane from z_{IR} to z. Then, as long as z is still far from the UV brane $z \gg 1$, we see that

$$A = v_{UV}, \qquad B(z) = \frac{v_{IR}}{z^{\Delta_+}} \left(1 - (\frac{z}{z_{IR}})^{-m^2/4} \right) . \tag{3.15}$$

This means that the UV coupling is begin kept fixed, but the theory is in a state $|\psi_z\rangle$, different from the vacuum, where $B = \langle \psi_z | O_\phi | \psi_z \rangle \neq 0$ differs from the vacuum expectation value of O_ϕ, $\langle O_\phi \rangle = 0$. Holding the IR brane at a position z corresponds to minimizing $\langle \psi | H_{CFT} | \psi \rangle$ over all states $|\psi\rangle$ subject to the constraint that $\langle \psi | O_\phi | \psi \rangle = B(z)$. Following standard arguments, this leads us to identify the radion with the Legendre transform of the source for O_ϕ, or put another way, the radion is the interpolating field for the operator O_ϕ. We recall this standard argument here for completeness. Suppose we want to minimize $\langle \psi | H | \psi \rangle$ subject to the constraint that $\langle \psi | O | \psi \rangle = \phi$. We can do this by introducing a Lagrange multiplier J and minimize

$$\min_{|\psi\rangle, J} \langle \psi | H | \psi \rangle - J(\langle \psi | O | \psi \rangle - \phi) = \min_{|\psi\rangle, J} \langle \psi | (H - JO) | \psi \rangle + J\phi \,. \tag{3.16}$$

Let first minimize w.r.t. $|\psi\rangle$, which only enters the first term. This is minimized when $|\psi\rangle = |O_J\rangle$, which is the ground state of the hamiltonian $H_J = H - JO$. The ground state energy is nothing other than $W(J)$. So, now, we have to minimize

$$\min_J \left(W(J) + J\phi \right) . \tag{3.17}$$

The minimum occurs at some value for the source J_ϕ, and the value of the function at the minimum is nothing other than the Legendre Transform $\Gamma(\phi)$. So, we have learned that the minimum value of the energy $\langle H \rangle$ subject to the constraint $\langle O \rangle = \phi$ is $\Gamma(\phi)$, and the state for which this minimum is attained is $|0_{J_\phi}\rangle$, which is the *ground* state of a *different* hamiltonian $(H - J_\phi O)$.

3.5 4D quantum gravity at M_{Pl}

We have understood from both the 5D gravity and 4D broken CFT points of view that there is interesting strongly coupled physics at the TeV scale in RS1. However, it is equally apparent from both sides that 4D quantum gravity does not become important until we reach energy scales of order M_{Pl}. So, all the physical questions involving strong 4D gravity, such as the resolution of 4D black-hole singularities or the big-bang singularity, can only be answered at M_{Pl}. This runs counter to the naive expectation from the 5D picture that the "strings" we see at the TeV brane are just red-shifted from the "strings" we would see on the Planck brane, which would lead us to conclude that we *can* learn about 4D quantum gravity at M_{Pl} by probing the theory at a TeV. However, this intuition is incorrect because although there do exist string modes localized to the TeV brane, just as the zero mode graviton is localized on the Planck brane, with exponentially small wave function on the TeV brane, all the rest of the states responsible for making 4D gravity finite are also localized on the Planck brane and are therefore inaccessible at TeV energies. This point is also quite clear in the string realization of these models along the lines of [15]. There, it is true that the QCD strings and the strings responsible for making gravity finite are in fact the same type IIB string. However, physics near the TeV and Planck branes probe the behavior of this IIB string on very different backgrounds. Close to the TeV brane, we are probing IIB string theory on the (deformed by the IR brane) AdS geometry, where it is indistinguishable from a QCD string. On the Planck brane, on the other hand, we are probing the IIB string on an essentially flat (compact) background, where it does *not* look like a QCD string, but the more conventional string theory in flat space.

3.6 Effective field theory

Finally we wish to make a comment on the effective field theory of these warped geometries. From the gravity side, at every z the theory is weakly coupled beneath the local cutoff $M_5(L/z)$ (in a string theoretic setting, the local cutoff would actually be $M_s(L/z)$, where M_s is the string scale). Furthermore, because of the redshift factor, all processes which start within the domain of validity of this effective theory remain within the effective theory. So we are free to write down any sort of effective theory we wish; for instance we can put SM fields on the TeV brane, or move the fermions around in different places in the bulk etc. However, not every effective theory can be consistently embedded into a full theory that has a sensible interpretation above a TeV. Normally, from a bottom-up point of view we do not think of this as being too constraining, since the detailed structure of the underlying "theory of everything" is not known. As long as the low energy theory is consistent, we can imagine that it will somehow be embeddable into the full theory. Of course, our lack of knowledge about the full theory means that we can't even in principle answer questions about physics at energies far above the cutoff. However, in the case where the effective theory is that of weakly coupled gravity on the AdS_5 geometries, AdS/CFT gives us a non-perturbative definition of the gravity theory, and tells us that the "theory of everything" that the effective theory gets embedded into is a strong 4D CFT. Therefore, while from the 5D point of view it seems perfectly innocuous to put SM fields on the TeV brane in the effective theory of RS1, we understand that embedding this picture into the fundamental theory requires us to find a broken CFT where the SM fermion, gauge boson and Higgs fields emerge as composites.

4. Gauge fields in the bulk

4.1 Phenomenology

Several groups have considered putting the SM gauge fields in the bulk of the RS1 geometry. The phenomenology of these models have a number of peculiar features quite different from the case of just gravity in the bulk. For instance, when the brane separation is chosen to generate the hierarchy, the wave function of the KK gauge bosons on the Planck brane is sizable, about $.2\times$ SM gauge couplings, while the KK graviton wave functions are strongly suppressed there. The KK gauge boson wave function on the TeV brane are a factor of 8 larger than the gauge coupling, with consequently severe limits on the mass of the first KK gauge boson in excess of 10 TeV if the SM fermions live on the TeV brane. In an effort to circumvent this problem, [23] considered the SM fermions localized away from the TeV brane at different points in the bulk, with the Higgs still on the TeV brane to maintain the solution to the hierarchy problem. The small overlap between the wave function of the left/right handed and Higgs fields helps generate a fermion mass hierarchy as in [24].[1]

Pomarol [25] considered the case of an SU(5) gauge theory broken to the SM in the bulk, with all the SM fields matter fields on the Planck brane. (Of course this now requires SUSY for the hierarchy problem, but if SUSY breaking is triggered on the IR brane then the warp factor can still be used to generate an exponential hierarchy, with the SUSY breaking transmitted to the matter fields via the SM gauge interactions.) The zero modes of the X, Y gauge bosons pick up a mass $\sim M_{GUT}$, while the KK excitations of the X, Y are at $\sim$ TeV. Furthermore, unlike the SM KK gauge boson excitations, the X, Y zero and KK modes have exponentially

[1]Actually in order to avoid FCNC problems from non-universal couplings to the KK gauge bosons, these KK modes must either be made heavier than ~ 100 TeV or the first two generations cannot be split, and the first-second generation hierarchies must come from a separate source [24].

suppressed wave functions on the TeV brane, and so give no problems with proton decay. However, since the X, Y KK modes are charged under the SM, they can be pair-produced at energies $\sim$ TeV. Pomarol also computed the renormalization of the SM gauge couplings, finding the interesting result that despite the presence of the fifth dimension and KK gauge bosons near a TeV, the gauge couplings continue to run logarithmically as in 4D, unifying at M_{GUT} in the usual way. However, this is not quite true. In fact, we will see there are large threshold corrections. This can again be seen on both sides of the duality.

As we will see, all of the above scenarios have a very simple holographic interpretation that greatly clarifies the physics. We summarize the main results here and present technical details in the next two subsections. The key point is that holographic dual of having gauge fields in the bulk with gauge group G is a broken 4D CFT, where a subgroup G of the global symmetry G_{gl} of the CFT has been weakly gauged. This is much as in the SM, where $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ weakly gauges a subgroup of the $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_V$ global symmetry of QCD, with leptons added as spectator fermions to cancel anomalies. In the simplest set-up, the Landau pole for the 4D gauge coupling is at the cutoff $1/L$ of the CFT, and it runs logarithmically to weaker values at lower energies.

Just as KK gravitons are spin 2 bound states of the broken CFT, the KK gauge bosons are spin 1 bound states. More specifically, whatever representation R of G_{gl} the CFT matter fields Q fall into, there will be $(Q\bar{Q})$ bound states transforming as the adjoint of G_{gl} and in particular as adjoints under the G subgroup. The bound states of this sort which have spin 1 are the KK gauge bosons. If the SM fermions are taken to live on the TeV brane, then they are composites of the broken CFT, and therefore their couplings to the spin 1 bound states are unsuppressed, surviving even in the limit where the SM gauge couplings are sent to zero. This provides a natural explanation for the stringent limits on this scenario found in [31].

The dual description of the scenario of [23], where the SM fermions are localized away from the TeV brane, but with the Higgs localized on the TeV brane, is similar in some respects to " walking extended technicolor" [26]. In contrast with RS1, where the SM gauge bosons must be thought as bound states of the broken CFT, here we have a situation where the SM is weakly gauged into a strong sector. Having the Higgs on the TeV brane is tantamount to saying that the strong sector spontaneously breaks some of its global symmetries, (the SM subgroup of which are weakly gauged). But this is exactly what we mean by technicolor. It is like "walking" technicolor because the strong dynamics has a large window where it is nearly conformal. However, in true walking technicolor, only some operators scale so slowly, whereas there is no such distinction here. Finally, a fermion mode localized in the bulk has the interpretation of a Higgsing of the high energy CFT, giving at low energies the fermion and another CFT. Integrating out the modes that become heavy through the Higgsing generates higher-dimension operators between the fermions and the low-energy CFT, and when the low energy CFT finally triggers electroweak symmetry breaking the fermions acquire their masses. This is essentially an *extended* technicolor theory. The difference is that the standard model fermions here are conformal field theory bound states. Furthermore, it is difficult to see how to generate generational mixing in this way.

The GUT scenario of [25] also has a simple holographic interpretation. We have a broken CFT with an SU(5) global symmetry, the SM subgroup of which is gauged at low energies. (We could imagine that the full SU(5) was gauged and broken at the GUT scale, with the real X, Y gauge bosons picking up a mass M_{GUT}.) Despite the fact that the CFT is strongly coupled, the 4D gauge couplings continue to run logarithmically far above the TeV scale. This is analogous to the fact that the QED gauge coupling runs logarithmically despite the coupling to quarks which feel the strong confining dynamics of QCD. As we will see, in the CFT case we can exactly compute and reproduce the logarithmic running despite the strong coupling, since the relevant $\langle jj \rangle$ correlator is fixed by conformal invariance. From this perspective, it would seem that the unification of couplings is preserved since the CFT states come in complete SU(5) multiplets. However, the number of GUT representations is large, as it is determined by N which is large if we are in the supergravity regime. So we expect unification will not survive this scenario. That is, there should be additional threshold corrections from the gravity picture, which are proportional to L/g_5^2. Because g_5^2 acts as a cutoff suppressing higher dimensional operators, one must require L large compared to this scale, so that we expect large threshold effects.

Since the CFT has an SU(5) global symmetry, there will be spin 1 resonances falling into the **24** of SU(5). These decompose under the SM gauge group as adjoints (the SM KK modes), and states with X, Y gauge boson quantum numbers (the X, Y KK modes). The CFT picture gives a simple explanation for the puzzling fact that the SM fermions have large amplitude for producing SM KK modes on the Planck brane, while the amplitude for producing X, Y KK modes and KK gravitons is exponentially suppressed. From the CFT point of view, the SM fermions interact with the CFT either indirectly through 4D gravity and the SM gauge interactions, or directly via higher-dimension operators suppressed by M_{Pl}. Since all the direct couplings between the SM fermion and the CFT are very suppressed, how can we get a sizeble amplitude for producing KK gauge bosons, which are after all bound states of the CFT? The answer is these spin 1 bound states are produced through mixing with the SM gauge bosons. This is like the electroproduction of the ρ meson in QCD: there is no direct coupling between electrons and quarks, but the ρ can be produced via its mixing with the photon, with an amplitude suppressed only by e_{QED}. Similarly in our case, the SM KK gauge bosons can be produces with an amplitude only suppressed by SM gauge couplings. On the other hand, the spin 1 bound states with X, Y quantum numbers *can not* mix with the SM gauge bosons because they do not have the same quantum numbers, and so the amplitude for singly producing them can only come from $1/M_{Pl}$ effects. They can however obviously they can be pair produced with SM gauge coupling strength.

So it appears that one can almost obtain a consistent grand unification scheme. However, one has to contend with large threshold effects which seem to destroy this unification.

4.2 Localization of Bulk Gauge Fields

The presence of gauge fields in the 5-d bulk gives rise to new interesting phenomena. Some of the material in this section has also been discussed in [27], and resulted from discussion with E. Witten.

In this section the metric is

$$ds^2 = \frac{L^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)\,, \qquad L \leq z \leq 1/\mu\,. \tag{4.1}$$

The Planck brane is located at $z = L$, while $z = 1/\mu$ is the position of the TeV brane: $\mu = O(1\,TeV)$.

By giving Neumann boundary conditions to the gauge field $\hat{A}_m$, one finds a discrete KK tower of states that begins with a massless mode, constant in z: $\hat{A}_5 = 0$, $\hat{A}_\mu(z,x) = A_\mu(x)$. Substituting this zero mode into the canonical action of a 5-d gauge field, with 5-d gauge coupling g_5, one finds

$$\begin{aligned}\frac{1}{4g_5^2}\int_L^{1/\mu} dz \int d^4x\sqrt{-g}g^{mp}g^{nq}F_{mn}(\hat{A})F_{pq}(\hat{A}) &= -\frac{1}{4g_5^2}\int_L^{1/\mu} dz \int d^4x \frac{L}{z}F^2_{\mu\nu}[A(x)] \\ &= -\frac{L}{4g_5^2}\log\mu L \int d^4x F^2_{\mu\nu}[A(x)]. \end{aligned}\tag{4.2}$$

The 4-d coupling constant that one reads off this formula is

$$g^2 = \frac{g_5^2}{L\log\mu L}\,. \tag{4.3}$$

Since g depends logarithmically on the TeV scale μ it seems natural to interpret eq. (4.3) as a running coupling constant evaluated at the infrared scale μ. Notice that this is the "standard" KK recipe if we identify μ with the compactification scale. Interpreting eq. (4.3) as the IR 4-d coupling constant is also consistent with — indeed, required by — holographic duality.

In our case the correct holographic interpretation is that an RS1 background with a propagating 5-d gauge field is dual to a 4-d CFT broken at the scale μ, coupled to a 4-d gauge field. More precisely, the gauge field couples to some conserved current of the broken CFT with running coupling constant $g(p)$. The coupling at $p = \mu$ is given by eq. (4.3), and it has a Landau pole at $p = O(1/L)$. The beta function coefficient b_{CFT} is identified as

$$\frac{b_{CFT}}{8\pi^2} = \frac{L}{g_5^2}\,. \tag{4.4}$$

Of course, below $p = \mu$, the coupling no longer runs.

To check this interpretation we compare the Green function for two sources located at the boundary $z = L$, computed in [25] with the propagator of the 4-d gauge field $\tilde{A}$, computed using the holographic duality.

Before doing this, let us remark that the 5-d gauge field action is non renormalizable, so that it makes sense only when dimensionful coupling constant g_5^2 is smaller than the cutoff length L; therefore, the 5-d description makes sense only when $L/g_5^2 \gg 1$.

4.3 Computation of the propagator

In the case where there the IR brane is removed, it is trivial to compute the full gauge boson propagator from the CFT side, essentially copying what was done for the case of just gravity in the bulk. The correction to the 4D gauge boson propagator is schematically

$$\frac{1}{p^2}\langle j(p)j(-p)\rangle\frac{1}{p^2}\,, \tag{4.5}$$

where j is the CFT current coupling to the gauge boson. Despite the strong coupling, the p^2 dependence of the $\langle jj\rangle$ correlator is fixed by conformal invariance, since j is a conserved current and therefore has vanishing anomalous dimension. We therefore have

$$\langle j(p)j(-p)\rangle \sim p^2\log p^2 \tag{4.6}$$

which yields a correction to the gauge boson propagator

$$\frac{\log p^2}{p^2} \tag{4.7}$$

giving precisely the logarithmic running of the gauge coupling. This confirms the result from the previous subsection: despite the appearance that there is no zero mode for the gauge field when the IR brane is removed (since the zero mode wave function is flat and the mode is nonnormalizable) , in fact we have a 4D gauge field coupled to a CFT, causing the gauge coupling to run logarithmically to zero in the IR.

We can do a precise calculation of the propagator, even in the presence of the IR brane, using the prescription of [2, 3]. First of all, we must solve the 5-d equations of motion of the field $\hat{A}_m$. This is most easily done by choosing the gauge $\hat{A}_5 = 0$, and by Fourier-transforming the fields $\hat{A}_\mu$ with respect to the coordinates x^μ. Defining $\tilde{A}_\mu(z,q) = \int d^4x\exp(iqx)\hat{A}_\mu(z,x)$ we find:

$$\begin{aligned}\tilde{A}_\mu(z,q) &\equiv C_\mu(q)\tilde{A}(q,z) = C_\mu(q)[bqzJ_1(qz) + qzY_1(qz)]\,,\\ \partial_z\tilde{A}_\mu(z,q) &\equiv C_\mu(q)\partial_z\tilde{A}(q,z) = qC_\mu(q)[bqzJ_0(qz) + qzY_0(qz)]\,.\end{aligned} \tag{4.8}$$

$C_\mu(q)$ and b are constant. The boundary condition at $z = 1/\mu$ is $\partial_z\tilde{A}_\mu(z,q) = 0$; it sets $b = -Y_0(q/\mu)/J_0(q/\mu)$.

At this point, we use the standard holographic correspondence [2, 3], summarized in eq. (1.1), to compute the self energy of the gauge field. Concretely, we interpret the 5-d gauge field at $z = L$ as a source for a conserved current of the broken CFT, and its on-shell 5-d action as the generating functional of theconnected Green functions. For the two-point function we have, with obvious notation

$$\begin{aligned}\langle J_\mu(0)\tilde{J}_\nu(q)\rangle &= \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Sigma(q);\\ \Sigma(q) &= \frac{1}{g_5^2}\left.\frac{\partial_z\tilde{A}}{\tilde{A}}\right|_{z=L}\\ &= q\frac{1}{g_5^2}\frac{Y_0(qL)+bJ_0(qL)}{Y_1(qL)+bJ_1(qL)}\\ &\approx q^2\frac{L}{g_5^2}\left[\log(qL/2)+\gamma-\frac{\pi Y_0(q/\mu)}{2J_0(q/\mu)}\right].\end{aligned} \tag{4.9}$$

The last approximate equality is valid when $qL \ll 1$.

To find the propagator of the dynamical gauge field, we resum the self-energy in the usual manner. Call e the coupling of A_μ to the broken CFT. Define $\langle A_\mu(0)\tilde{A}_\nu(q)\rangle = (\eta_{\mu\nu} - q_\mu q_\nu/q^2)\Pi(q)$; then

$$\Pi^{-1}(q) = [q^2 - e^2\Sigma(q)] = q^2\left\{1 - \frac{e^2L}{g_5^2}\left[\log(qL/2)+\gamma-\frac{\pi Y_0(q/\mu)}{2J_0(q/\mu)}\right]\right\}. \tag{4.10}$$

Let us consider at first large euclidean momenta $q \to iq$, $|q| \gg \mu$. In this case, $\pi Y_0(iq/\mu)/2J_0(iq/\mu) \approx i\pi/2$ and the resummed propagator produces a *logarithmically running* coupling constant

$$\frac{1}{e^2_{\text{eff}}(q)} = \frac{1}{e^2} - \frac{L}{g_5^2}[\log(qL/2)+\gamma]. \tag{4.11}$$

This result agrees with ref. [25] when $e \to \infty$, i.e. when the Landau pole is at $2\exp(-\gamma)/L$, i.e. at an energy of the order of the UV cutoff $1/L$. Notice that the running is logarithmic at energies well above the KK scale μ. This effect is easy to understand in the 4-d field theory dual of the RS compactification, since this dual is a broken CFT weakly coupled to a gauge field.

The explicit computation of the propagator has shown that our interpretation of the coupling constant given in eq. (4.3) as an IR coupling, evaluated at the scale μ is correct. In particular, even in the cutoff CFT limit $\mu \to 0$, the gauge field does not decouple, since only the IR coupling vanishes, not the propagator at nonzero q. True decoupling is obtained by removing the UV brane, not by removing the IR brane.

Finally, notice that the position of the Landau pole can be changed by adding a 4-d boundary term $(1/4e_0^2)\int d^4xF^2_{\mu\nu}(A)$ to the 5-d action in eq. (4.2).

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4.4 KK gauge boson production

One of the intriguing results of ref. [25] is that if on the UV brane there exist fields charged under the 5-d gauge field, the amplitude for direct production of KK gauge bosons is non-negligible. Superficially, this seems in contradiction with the holographic interpretation. In Pomarol's model, indeed, the standard model fields live on the UV brane, instead of living on the TeV brane, as in RS1. In the dual 4-d field theory,the KK gauge bosons are bound states of the broken CFT, and there is no direct coupling between the "standard model" fields, living as we just said on the UV brane, and the CFT. All interactions between these two sectors are mediated either by a 4-d graviton or by a 4-d gauge field. Graviton-mediated interactions are negligible at energies below $1/L$. On the other hand, kinetic-term mixing between the gauge field and the CFT spin-1 bound states may (and indeed does) account for Pomarol's result.

First of all, let us find the spectrum of KK excitations. A KK state obeys Neumann boundary conditions both at $z = 1/\mu$ and $z = L$. This is consistent with the holographic interpretation of KK excitations as bound states of the broken CFT [28]. The b.c. at $z = 1/\mu$ gave $b = -Y_0(qL)/J_0(qL)$; the b.c. at $z = L$ implies

$$J_0(q/\mu)Y_0(qL) = J_0(qL)Y_0(q/\mu)\,. \tag{4.12}$$

For $\mu L \ll 1$, this equation is approximately solved by $J_0(q/\mu) = 0$; therefore, the spectrum of KK excitations is discrete and quantized in units of μ.

By coupling the broken CFT to the gauge field A_μ, the mass of the KK excitations/bound states is given by the position of the poles in the propagator $\Pi_{\mu\nu}(q)$, and it is shifted from the $e = 0$ value by terms $O(e^2)$. The mass of the n-th bound state is then $m_n = \mu j_{(0,n)} + O(e^2)$; $j_{(0,n)}$ is the n-th zero of J_0.

The n-th spin-1 bound state couples to Planck brane ("Standard Model") fields with strength F given by

$$F^2 = 2m_n \mathrm{Res}\, e^2\Pi(q)|_{q\approx m_n}\,. \tag{4.13}$$

The residue is most easily evaluated by rewriting $e^2\Pi$, using definition (4.11), as

$$e^2\Pi(|q|) = \frac{e^2_{\text{eff}}(|q|)}{q^2}\left[1 + e^2_{\text{eff}}(|q|)\frac{\pi L}{2g_5^2}\frac{Y_0(q/\mu)}{J_0(q/\mu)}\right]^{-1}\,. \tag{4.14}$$

By expanding the expression in brackets in powers of e^2_{eff}, and thanks to the Bessel function identities $J_0'(j_{(0,n)}) = -J_1(j_{(0,n)})$ and $J_1(j_{(0,n)})Y_0(j_{(0,n)}) = 2/\pi j_{(0,n)}$ we find that to order e^2_{eff}, the coupling F is

$$F = \sqrt{\frac{2L}{g_5^2}}\frac{e^2_{\text{eff}}(m_n)}{(m_n/\mu)J_1(m_n/\mu)}\,. \tag{4.15}$$

This formula reduces exactly to the one found by Pomarol in [25] when the Landau pole is set at $2\exp(-\gamma)/L$, i.e. when $e \to \infty$.

5. Probe branes

In [29, 30], models for solving the hierarchy problem with infinitely large dimensions were considered, where the SM fields are localized on a probe 3-brane away from the brane where gravity is localized. Specifically, in [30], there is an infinite AdS_5 with a 3-brane located away from the Planck brane where the SM is taken to live, with the warp factor generating the TeV scale exponentially as in RS1. Since we have an AdS both to the left and the right of the probe brane, the 4D picture must be that at low energies we have $SM \times CFT_L$, which merge into a single CFT_H theory at energies above $\Lambda_{IR} \sim$ TeV. An example of this scenario is realized if, for instance, in the Standard $\mathcal{N} = 4$ case we start with $N+1$ branes and remove a single "probe" brane away from the remaining stack of N branes. From the gravity side we will have something that looks identical to LR (except without the Planck brane). From the 4D viewpoint, we have just Higgsed $\mathrm{U}(N+1) \to \mathrm{U}(N) \times \mathrm{U}(1)$ at Λ_{IR}. Now, LR computed the correction to Newton's law on the probe brane, finding

$$V(r) \sim \frac{m_1 m_2}{M_4^2}(\frac{1}{r} + \frac{L^2}{r^3}) + \frac{m_1 m_2}{TeV^8 r^7}\,, \tag{5.1}$$

where the last term is new and dominates at short enough distances. This can be understood from the 4D point of view as follows. After integrating out massive states at the scale Λ_{IR}, we are left with irrelevant operators linking the SM fields to the fields of the low-energy CFT_L. An operator allowed by all possible symmetries is

$$\frac{1}{\Lambda_{IR}^4} T^{\mu\nu}_{SM} T_{\mu\nu\, CFT_L}\,. \tag{5.2}$$

Exchanging the CFT_L states then generates the operator

$$\frac{1}{\Lambda_{IR}^8} T^{\mu\nu}_{SM}(p) \langle T_{\mu\nu\, CFT_L}(-p) T^{\rho\sigma}_{CFT_L}(p) \rangle T_{\rho\sigma\, SM}(-p) = \frac{c p^4 \log p^2}{\Lambda_{IR}^8} T^{\mu\nu}_{SM}(p) T_{\mu\nu\, SM}(-p)\,, \tag{5.3}$$

which precisely yields the $1/(TeV^8 r^7)$ non-relativistic potential found by LR. Here we have just posited the existence of the $T^{\mu\nu}_{SM} T_{\mu\nu\, CFT}$ operator, and showed that it would reproduce the LR result, but in the $\mathcal{N} = 4$ SYM case we can prove its existence. If we go out along the Coulomb branch, Higgsing $\mathrm{U}(N+1) \to \mathrm{U}(N) \times \mathrm{U}(1)$, we can integrate out the heavy states and obtain the low-energy action. Of course, the theory is strongly coupled, but some quantities, in particular the $\mathrm{Tr}\, F^4$ piece of the effective action, are known to be 1-loop exhausted by a nonrenormalization theorem. These operators have the same dimension as $T^{\mu\nu}_{\mathrm{U}(1)} T_{\mu\nu\, \mathrm{SU}(N)}$, and so in this specific case we can explicitly see that dimension-8 operators are generated.

We can also consider the case of bulk gauge fields. Of course, in this case these be the SM gauge fields, since we can not phenomenologically tolerate the gapless CFT states charged under the SM gauge group. However, we can imagine some other gauge field in the bulk, say coupled to $B - L$, under which the SM fermions are charged.

In the case of pure gravity in the bulk, the cross-section for KK graviton production was $\sigma \sim E^6/\Lambda_{IR}^8$, which is the same as the case of $n = 6$ large extra dimensions. Indeed, we even expect all differential cross-sections to be the same as in the $n = 6$ case, since in both cases the cross-section for graviton production is related to the imaginary part of the graviton exchange diagram, and this graviton exchange propagator is the same $1/x^8$. When there are gauge fields in the bulk, however, the cross-section for producing KK gauge bosons is much larger. The reason is that we can produce CFT modes directly through s channel production. The cross-section is

$$\sigma(s) \sim \frac{L}{g_5^2} \frac{e_{\text{eff}}^4(s)}{s}, \tag{5.4}$$

and since there is no gap, this is deadly phenomenologically unless the 4D gauge coupling e_{eff} is tiny. This can be done either if the β function is huge or if a large boundary gauge coupling is added on the Planck brane.

6. Inflation and cosmology

The 4D holographic viewpoint is particularly useful for thinking about the early universe cosmology of models with AdS_5 slices. The late cosmology of RS2 has been discussed in e.g. [32]. The early cosmology of RS2 with a heated bulk has been addressed in [7]. Let us turn to RS1. As we have discussed, contrary to the naive expectation that we cannot extrapolate cosmology to temperatures above $\sim$ TeV because of "hitting string theory at a TeV", at temperatures higher than TeV the universe has normal radiation-dominated FRW expansion from a hot CFT. In the RS2 case, there is a solution of the 5D gravity equations which has an induced 4D metric on the Planck brane given by a radiation dominated FRW cosmology, with a retreating horizon in the bulk [7]. Physically, this corresponds to the bulk initially heated up near the Planck brane, with the hot region gradually expanding away and cooling down. The cosmology also follows from the analysis of [33], who considered other potential cosmological solutions in and AdS-Schwarschild background, and found the brane moves through the static background in accordance with the expected four-dimensional behavior.

Now we consider a theory which at zero temperature has an infrared breaking of the CFT, to generate a TeV brane. We now ask what this looks like at early times. From the holographic viewpoint, it is clear that one does not expect to see a TeV brane at temperatures well above a TeV, since the ultimate breaking of the conformal field theory applies at temperatures below that scale. So at high temperatures, there are two possible scenarios. The first is that the TeV brane simply does not exist at early times; the horizon shields the region where the TeV brane might exist. As the temperature drops and the horizon receeds, it eventually uncloaks the TeV brane, and the true SM degrees of freedom emerge. An alternative possibility with similar physical consequences is that a TeV brane exists at the scale associated with the temperature of the theory. Only when the temperature drops to appropriately low scale will the brane settle at its true minimum, analogously to the behavior of other moduli in the early universe. Notice that in either case, if we distinguish the AdS curvature and the fundamental Planck scales, we expect a regime between these scales multiplied by the TeV warp factor in which the theory appears five, not four dimensional, as the TeV brane settles to its true minimum.

There has been some concern about whether inflation can be made to work in RS, while getting large enough density perturbations. The worry has been that one cannot put the inflaton on the TeV brane, since its energy density will be too low to generate sufficient density perturbations, and that putting it on the Planck brane would make it impossible to directly reheat SM fields. The CFT viewpoint shows to the contrary that doing inflation is standard; we just add an inflaton with $1/M_{Pl}$ couplings to the CFT. After inflating near the GUT scale to generate appropriate $\delta\rho/\rho$, the inflaton reheats the CFT, which eventually turns into SM particles when the temperature drops beneath a TeV. The bulk picture of all this is the following. After inflating, the inflaton on the Planck brane reheats the bulk in the immediate vicinity of the Planck brane. This is clear from the KK picture since it has unsuppressed couplings to the heaviest KK modes. This reheating generates a horizon in the bulk and starts us off in the FRW solution of [7]. The ensuing evolution is the same as described in the previous paragraph, and the SM is eventually reheated to $T \sim$ TeV. It would be interesting to investigate this 5D picture in more detail.

Much of the worry about the early universe cosmology in RS1 has evolved around the light radion. In particular, if we imagine that in the early universe the IR and UV branes were far displaced from their final equilibrium values, then the early cosmology will certainly not be standard FRW expansion. We can understand this from the 4D viewpoint given the holographic interpretation of the radion we discussed in the previous subsection. Moving the IR and UV branes say, much closer to each other, corresponds to putting the broken CFT in a very special, *non-thermal* state. It is not surprising that such a state should yield unusual cosmology. If the initial condition for QCD was a coherent state where e.g. $\mathrm{Tr}\,(F^2) = M_{GUT}^4$ then we would also expect unusual cosmology! However, we do not usually imagine such an initial state for the universe. Rather, we imagine that the early universe was in a hot thermal state, and indeed this can come about through standard inflation and reheating. So, while it is perfectly fine to consider cosmology with widely varying radions as a mathematical exercise, this does not correspond to realistic initial conditions for early cosmology. The more reasonable initial condition is a thermal one, where from the 5D point of view the TeV brane is dissolved, and instead there is a horizon away from the Planck brane, giving the FRW solution of [7].

The cosmology of the probe brane scenarios is also interesting. From the CFT point of view, it is clear that the reheating temperature can not exceed $\sim$ TeV. If this happens, then as the high energy CFT cools down beneath a TeV, the energy density get divided into the SM and the low-energy CFT, which is still a large N gauge theory redshifting as radiation. This would give too many degrees of freedom during nucleosynthesis. Therefore, we have to imagine that only the SM degrees of freedom are reheated. This might be difficult, since the coupling to the standard model states at high energy corresponded to a coupling to the full high energy CFT. It is difficult to see how something could decay into standard model states but not the low-energy CFT states. Here, however, we will take the most conservative viewpoint and assume that initially only the SM fields are heated up. We then demand that they do not heat up the low energy CFT.

With just gravity in the bulk, the low energy CFT is coupled to the SM most strongly only via the dim. 8 operator TT operator, and so the cross section for producing the CFT states with the SM fields at temperature T is $\sigma \sim T^6/\Lambda_{IR}^8$. In order not to thermalize the CFT, the mean free path $l \sim (n\sigma v)^{-1}$ should be much larger than the Hubble size $H^{-1} \sim M_{Pl}/T^2$, and therefore we have the constraint

$$\frac{T^9}{\Lambda_{IR}^8} \lesssim \frac{T^2}{M_{Pl}} \rightarrow T \lesssim 10\text{GeV} \left(\frac{\Lambda_{IR}}{\text{TeV}}\right)^{8/7} . \tag{6.1}$$

It is interesting that this constraint on the reheating temperature is similar to the case of large extra dimensions with $n = 6$ [35]. However the physics is quite different and the constraint on the probe brane theories are actually somewhat weaker. In the large dimension case, the KK gravitons are produced through a process of evaporation: since each individual KK mode only has $1/M_{Pl}$ interactions, while copious amounts of energy can be lost into many KK modes, once produced each individual one hardly interacts with the others or with the SM fields except over huge timescales comparable or longer than the present age of the universe. Furthermore, they red-shift away as massive particles, so once they are produced there is a greater danger that they may come to dominate the energy density of the universe. The most severe constraint came from the long-lived decays of the massive KK gravitons to e.g. photons or e^+e^- pairs [35, 36]. None of these things happen in the probe brane case, since it is clear from the 4D viewpoint that the energy thrown into the CFT will redshift as radiation, and so as long as the nucleosynthesis constraint is obeyed there is no worry about overclosure or late decays.

If there are gauge fields in the bulk, then unless the 4D gauge coupling is tuned to be tiny (by adding a large boundary gauge coupling on the Planck brane), the CFT is immediately thermalized with the SM by the gauge interactions. The CFT and SM thermalize at a temperature when

$$\frac{L}{g_5^2} e_{\text{eff}}^4(T) T \sim \frac{T^2}{M_{Pl}} \rightarrow T = \frac{L}{g_5^2} e_{\text{eff}}^4(T) M_{Pl} \, . \tag{6.2}$$

Requiring that this not happen till after BBN where $T \sim$ MeV places the constraint

$$\left(\frac{L}{g_5^2}\right)^{1/4} e_{\text{eff}}(\text{MeV}) \lesssim 10^{-5} \,. \tag{6.3}$$

7. RS and ADD

We summarize with a comparison of the similarities and differences between the RS1 and ADD large extra dimensions [34] scenarios. In both theories, one can view TeV as the fundamental scale for observers on the brane, with the Planck scale being an induced scale. In the ADD scenario, that is the only point of view; the fundamental scale of quantum gravity is a TeV; the Planck scale is a result of the small coupling of a graviton spread over a large volume to the fields on the brane. In the RS1 scenario on the other hand, it is only observers on the TeV brane who see the TeV scale as the quantum gravity scale. The shape of the KK modes (and the string modes) is such that the scale for 5D quantum gravity (which we have seen is equivalent to a conformal field theory) depends on the location in the fifth dimension. Nonetheless, if we live on the TeV brane, we see strong interactions at a TeV, and KK modes at a TeV. However, we do not see the string modes associated with the 4D Planck scale.

In this sense, RS1 is a theory which has both the TeV scale and the Planck scale with a desert in between. Of course, the theory is strongly interacting at the TeV scale. Nonetheless, cosmologically and otherwise, one can ask questions pertaining to physics above the TeV scale. Indeed, as we have discussed these models have a holographic 4D description by a conventional *local* field theory all the way up to the scale $1/L \lesssim M_{Pl}$. This field theory is strongly coupled and conformal above a TeV. The solution to the hierarchy problem using the exponential warp factor from the 5D picture is translated into dimensional transmutation in the 4D description.

This being said, the strong coupling field theory gives rises to many interesting physical phenomena near a TeV, and there is a window of energies, where the 5D gravity description is weakly coupled, where these are most usefully thought of as "5D TeV quantum gravity effects". The window of energies for which this 5D description is useful is given by $1/z < E < M_5L/z$ where z is the position of the IR brane. Since the mass of the first KK graviton is given by $M_{KK} = 1/z$, this window can be expressed as

$$M_{KK} < E < (M_5L)M_{KK} \,. \tag{7.1}$$

For $E < M_{KK}$, there is a weakly coupled 4D effective theory, while for $E > (M_5L)M_{KK}$ the gravity description is strongly coupled and the only non-perturbative definition of the theory is in terms of the 4D CFT description. For $M_5L \sim 1$ this window does not exist, but we can certainly imagine $M_5L \sim 10$ so we could have an order of magnitude in energy where the 5D gravity description is useful. Many interesting predictions follow from this picture, for instance, there are about (M_5L) KK gravitons starting at M_{KK}, which are well defined, narrow resonances. These are of interpreted as spin 2 bound states of the 4D broken CFT, but in the absence of putting the 4D theory on a computer their existence can only be inferred from the weakly coupled 5D gravity picture. Furthermore, black holes on the TeV brane with size between and $(M_5 L M_{KK})^{-1}$ and M_{KK}^{-1} are essentially "flat space" 5D black holes and are made with the usual cross-section. Again, these states can be interpreted as excitations of the CFT from the 4D viewpoint, but the properties of these states are most usefully described in the weakly coupled 5D picture. On the other hand, the cross-section for making black holes larger than M_{KK}^{-1} stops increasing and hits a maximum of $\sim$ TeV^{-2}. This is very different than the usual intuition for black hole production in flat space, where the cross-section grows indefinitely with energy. As we saw, this difference could be easily understood both from the gravity and the CFT pictures.

We have seen that processes around a TeV can be most conveniently thought of as arising from 5D quantum gravity effects, but as we have remarked RS1 is a theory with 4D quantum gravity at the Planck scale; therefore many of the effects we usually associate with "quantum gravity" do not manifest themselves at energies lower than M_{Pl}. For instance, big-bang cosmology is standard FRW cosmology for temperatures far above a TeV, and what resolves the big-bang singularity does not manifest itself before M_{Pl}. The resolution of 4D black-hole singularities occurs at M_{Pl}. 4D General Relativity is rendered a finite theory due to additional states that show up at M_{Pl}. This is of course obvious from the 4D description, because we have simply added 4D GR to a broken strong CFT, but unraveling the dynamics of the CFT tells us nothing about what eventually makes gravity a sensible theory above M_{Pl}. Therefore, the experimental observation of the "5D TeV quantum gravity effects" would shed no light on the 4D quantum gravity effects discussed above. From the 5D side, this can be seen because the closed string spectrum of states falls into two separate sectors, one near a TeV and the other near M_{Pl}.

By contrast, the large extra dimensions scenario has only a single quantum gravity scale $\sim$TeV^{-1}. *All* quantum gravity effects are associated with the higher-dimensional Planck scale, that can be as low as $\sim 1 - 10$ TeV, and all closed string states are at a TeV. These theories break the standard rules for building extensions of the SM (and also make the construction more challenging), since instead of modifying the theory starting at short distances $\sim$ TeV^{-1}, the theory is modified at distances much *larger* than a TeV^{-1} by the addition of large new dimensions, or equivalently by the addition of 10^{32} new states (the KK gravitons) *lighter* than a TeV. And finally, since the size of the extra dimensions in the ADD scenario can be as large as a few hundred microns, corrections to the $1/r$ newtonian potential start at almost macroscopic distances, unlike in RS, where they start at almost planckian distances $O(L)$.

One can ask what would be the experimental consequence of this distinction in theories. *If* experiments can reach the quantum gravity scale (which is above the scale for the KK modes), one would see quite different strong interactions. In particular, in the ADD scenario, one would see "conventional" black hole behavior (appropriate for larger dimensions), whereas in the RS1 scenario, there would only be a window of energies for which this applies, after which one sees cross sections most readily described by the 4D conformal field theory. In the ADD scenario, one would really be probing the fundamental gravity theory; this is not the case in the warped scenarios. One interesting consequence is that the LR theory [30] would be distinguished from the six large extra dimension scenario if and only if the quantum gravity effects are measured.

It should be noted that all the above conclusions about RS1 apply to theories in which the warp factor generates the hierarchy, whereas all the conclusions about ADD apply to theories in which large dimensions, and no warp factor, are responsible for the hierarchy. Interesting variants of both these have been considered, and in some sense, interpolate between the two different behaviors of quantum gravity. For instance, [29] considered intersecting brane scenarios with a large number of extra dimensions. In this case, one can generate a hierarchy by choosing a low scale of quantum gravity, or from the warp factor (or a combination of both). If the low scale of quantum gravity is chosen of order TeV, and our physical 3-brane is at the intersection of the branes, the phenomenology reproduces that of large extra dimension scenarios. On the other hand, if the hierarchy is produced by placing the 3-brane somewhere away from the intersection, then the fundamental gravitational scale will be higher, and 4D quantum gravity effects will not emerge at the TeV scale. On the other hand, [37] considered lowering the fundamental scale of the brane tension, and the 5D cosmological constant to a TeV, with the fundamental Planck scale around 10^5 TeV. In this case, one can still use the warp factor to generate the Planck scale. However, in this scenario, one has very light KK modes, in fact of order mm^{-1}! So here, the sub-millimeter phenomenology is similar to the large dimension scenario.

We conclude that TeV experiments have the potential to unravel the physics of extra dimensions, should we reach the higher dimensional gravitational scale.

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References

[1] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[5] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[6] J. Maldacena, unpublished;
H. Verlinde, invited talk at ITP Santa Barbara conference, *New dimensions in field theory and string theory*, http://www.itp.ucsb.edu/online/susy_c99/verlinde;
E. Witten, ibidem, http://www.itp.ucsb.edu/online/susy_c99/discussion.

[7] S.S. Gubser, *AdS/CFT and gravity*, *Phys. Rev.* **D 63** (2001) 084017 [hep-th/9912001].

[8] M.J. Duff and J.T. Liu, *Complementarity of the Maldacena and randall-sundrum pictures*, *Phys. Rev. Lett.* **85** (2000) 2052 [hep-th/0003237].

[9] S. Giddings, E. Katz and L. Randall, *Linearized gravity in brane backgrounds*, *J. High Energy Phys.* **03** (2000) 023 [hep-th/0002091].

[10] M. Porrati and A. Starinets, *Rg fixed points in supergravity duals of 4-d field theory and asymptotically AdS spaces*, *Phys. Lett.* **B 454** (1999) 77 [hep-th/9903085].

[11] V. Balasubramanian and P. Kraus, *Spacetime and the holographic renormalization group*, *Phys. Rev. Lett.* **83** (1999) 3605 [hep-th/9903190].

[12] J. de Boer, E. Verlinde and H. Verlinde, *On the holographic renormalization group*, *J. High Energy Phys.* **08** (2000) 003 [hep-th/9912012].

[13] E. Verlinde and H. Verlinde, *RG-flow, gravity and the cosmological constant*, *J. High Energy Phys.* **05** (2000) 034 [hep-th/9912018].

[14] M. Henningson and K. Skenderis, *The holographic Weyl anomaly*, *J. High Energy Phys.* **07** (1998) 023 [hep-th/9806087].

[15] H. Verlinde, *Holography and compactification*, *Nucl. Phys.* **B 580** (2000) 264 [hep-th/9906182].

[16] S.B. Giddings and E. Katz, *Effective theories and black hole production in warped compactifications*, hep-th/0009176.

JHEP08(2001)017

[17] P.H. Frampton and C. Vafa, *Conformal approach to particle phenomenology*, `hep-th/9903226`.

[18] C. Csaki, W. Skiba and J. Terning, *Beta functions of orbifold theories and the hierarchy problem*, *Phys. Rev.* **D 61** (2000) 025019 [`hep-th/9906057`].

[19] S.S. Gubser, I.R. Klebanov and A.W. Peet, *Entropy and temperature of black 3-branes*, *Phys. Rev.* **D 54** (1996) 3915 [`hep-th/9602135`].

[20] W.D. Goldberger and M.B. Wise, *Phenomenology of a stabilized modulus*, *Phys. Lett.* **B 475** (2000) 275, [`hep-ph/9911457`];
C. Charmousis, R. Gregory and V.A. Rubakov, *Wave function of the radion in a brane world*, *Phys. Rev.* **D 62** (2000) 067505 [`hep-th/9912160`].

[21] W.D. Goldberger and W.R. Wise, *Modulus stabilization with bulk fields*, *Phys. Rev. Lett.* **83** (1999) 4922, [`hep-ph/9907447`].

[22] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [`hep-th/9909134`].

[23] T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, *Nucl. Phys.* **B 586** (2000) 141 [`hep-ph/0003129`].

[24] N. Arkani-Hamed and S. Dimopoulos, *New origin for approximate symmetries from distant breaking in extra dimensions*, `hep-ph/9811353`;
N. Arkani-Hamed and M. Schmaltz, *Hierarchies without symmetries from extra dimensions*, *Phys. Rev.* **D 61** (2000) 033005 [`hep-ph/9903417`].

[25] A. Pomarol, *Grand unified theories without the desert*, *Phys. Rev. Lett.* **85** (2000) 4004 [`hep-ph/0005293`].

[26] B. Holdom, *Techniodor*, *Phys. Lett.* **B 150** (1985) 301.

[27] N. Kaloper, E. Silverstein and L. Susskind, *Gauge symmetry and localized gravity in M-theory*, *J. High Energy Phys.* **05** (2001) 031 [`hep-th/0006192`].

[28] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [`hep-th/9803131`].

[29] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, *Infinitely large new dimensions*, *Phys. Rev. Lett.* **84** (2000) 586 [`hep-th/9907209`].

[30] J. Lykken and L. Randall, *The shape of gravity*, *J. High Energy Phys.* **06** (2000) 014.

[31] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Bulk gauge fields in the Randall-Sundrum model*, *Phys. Lett.* **B 473** (2000) 43 [`hep-ph/9911262`].

[32] C. Csaki, M. Graesser, L. Randall and J. Terning, *Cosmology of brane models with radion stabilization*, *Phys. Rev.* **D 62** (2000) 045015 [`hep-ph/9911406`].

[33] P. Kraus, *Dynamics of anti-de Sitter domain walls*, *J. High Energy Phys.* **12** (1999) 011 [hep-th/9910149].

[34] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398];
N. Arkani-Hamed, L. Hall, D. Smith and N. Weiner, *Solving the hierarchy problem with exponentially large dimensions*, *Phys. Rev.* **D 62** (2000) 105002 [hep-ph/9912453].

[35] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and tev scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344].

[36] L.J. Hall and D. Smith, *Cosmological constraints on theories with large extra dimensions*, *Phys. Rev.* **D 60** (1999) 085008 [hep-ph/9904267].

[37] D.J.H. Chung, L.L. Everett and H. Davoudiasl, *Experimental probes of the Randall-Sundrum infinite extra dimension*, hep-ph/0010103.

JHEP08(2001)017